Syllabus | Aptitude test 2025

BSc Mathematical Foundations of Data Science | University of Vienna

This document has been provided by Univ.-Prof. Michael Eichmair, PhD.[1]
You can find many practice problems in line with this syllabus under the following link:
https://mmf.univie.ac.at/materialien/mathematik-auf-augenhoehe/

1 Algebra and geometry

1.1 Numbers, quantities, and sets

Students should be able to

- carry out operations with numbers from the number systems $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$; use the relationships between these number systems; interpret different representations of numbers and switch between them; interpret and set up interval notation; interpret and set up representations of numbers and sets of numbers on the number line;
- set up and interpret enumerative and descriptive representations of sets; switch between them; carry out and interpret the following set operations: intersection, union, difference, complement; set up and interpret Venn diagrams; interpret and use the mathematical symbols $\in$, $\notin$, $\subseteq$, $\{\ \}$, $\emptyset$, $\cap$, $\cup$, and $\setminus$ correctly
- determine the prime factorization of a positive integer; apply the relationship between the prime factorization and the divisor set of a positive integer; find the least common multiple and the largest common divisor of positive integers;
- represent quantities as a combination of numerical value and unit and interpret such representations; work with powers of ten and associated prefixes from 10^{-12} (pico) to 10^{12} (tera); work with floating point representations in the form $\pm a \cdot 10^k$ with $1 \leq a < 10$, $a \in \mathbb{R}$, $k \in \mathbb{Z}$;
- find and interpret relative proportions; use percent and per mille notation.

1.2 Terms, formulas, equations

Students should be able to

- set up, transform and interpret terms and formulas, also using basic geometry facts
- represent numbers using integer and rational exponents; interpret such representations of numbers and switch to different representations; apply the laws of exponents
- represent numbers using logarithms; interpret such representations of numbers; apply the laws of logarithms (the change of bases formula is not required); solve exponential equations;

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- set up and solve linear equations in one variable; interpret solutions;
- set up and solve quadratic equations in one variable; interpret solution cases and solutions;
- set up linear equations in two variables, transform such equations; graph solutions of such equations; set up systems of linear equations in two variables; transform such systems; solve such systems algebraically and graphically; interpret solution cases and solutions.

1.3 Vectors in $\mathbb{R}^2$ und $\mathbb{R}^3$

Students should be able to

- interpret and represent vectors geometrically as points or as arrows and conversely; carry out and interpret addition and subtraction of vectors as well as multiplication of a vector with a scalar; find and interpret the magnitude of a vector and the associated unit vector; solve geometric problems using vectors;
- find the dot product of two vectors; interpret the value of the dot product (positive, zero, negative); find the angle between two vectors; find vectors normal to a given vector (the cross product of two vectors $\mathbb{R}^3$ is not required);
- set up and interpret equations and parameter representations of straight lines in $\mathbb{R}^2$; graph straight lines described in these ways; switch between these representations of straight lines; set up and interpret parameter representations of straight lines in $\mathbb{R}^3$;
- justify whether a given point lies on a given line; determine the relative position of two given lines; find the intersection of two intersecting lines.

1.4 Trigonometry

Students should be able to

- apply the definitions of $\sin(\alpha)$, $\cos(\alpha)$, and $\tan(\alpha)$in right triangles and the unit circle; solve geometric problems using right triangles;
- apply the relationships between $\sin(\alpha)$, $\cos(\alpha)$, and $\tan(\alpha)$; solve equations of the form $\sin(\alpha) = c$, $\cos(\alpha) = c$, and $\tan(\alpha) = c$ for α graphically on the unit circle and, for special values of c, exactly;
- use percentages and ratios in a slope triangle to find and interpret the slope of a line; find and interpret the angle of slope of a line; switch between these representations.

2 Functional dependencies

2.1 Representations and properties of real functions

- Argue whether a given verbal, tabular, graphical or algebraic relationship may be represented by a function; recognize function types in formulas and sketch the graphs

of the associated function; in case of functions, switch between the aforementioned representations; find and interpret values and ordered pairs of a function;
- find and interpret measures of change of a function (absolute change, relative or percentage change, average rate of change);
- find the zeroes of a function whose term is given in product form; create and interpret tables of signs for such functions;
- know and apply the following properties of functions: zero, intersection with the vertical axis, symmetry with respect to the vertical axis or to the origin, horizontal and vertical asymptotes, periodicity, monotonicity, local and global extremum, convexity/concavity, turning points; read off such properties of a function from its graph;
- sketch the graph of a function with given properties;
- find graphically or algebraically where the graphs of two functions intersect; interpret such points of intersection;
- form new functions from a function f using a parameter c: $g_1(x) = c \cdot f(x)$; $g_2(x) = f(c \cdot x)$; $g_3(x) = f(x) + c$; $g_4(x) = f(x + c)$
- know and apply relationships between the graph of f and the graphs of g_1, g_2, g_3, g_4.

2.2 Linear functions: $f(x) = k \cdot x + d$

- Argue whether a given process can be modelled by a linear function; find and interpret the parameters k and d of a linear function; draw graphs of linear functions;
- know and use linear functions of the form $f(x) = k \cdot x$ to represent directly proportional relationships;
- know and use linear functions as models for growth and decay processes with $f(0) = d$ and $f(x + 1) = f(x) + k$.

2.3 Power functions: $f(x) = a \cdot x^z$ with $z \in \mathbb{Z} \setminus \{0\}$

- Recognize and sketch graphs of power functions;
- find the parameters a, c and the exponent z of functions of the type $f(x) = a \cdot x^z + c$; interpret the parameters a and c;
- know and use power functions of the type $f(x) = \frac{a}{x}$ as representations of indirectly proportional relationships.

2.4 Polynomial functions

Students should be able to

- interpret the value of c and the sign of a for quadratic functions f with $f(x) = a \cdot x^2 + b \cdot x + c$; complete the square and determine the vertex of such functions; apply the symmetry of quadratic functions with respect to the vertex;
- apply the relationship between the degree n of a polynomial function and the possible number of its zeroes, stationary points, and turning points; recognize the graphs of polynomial functions and their possible degrees; sketch the graphs of polynomial functions;

- apply the representation $g(x) = a \cdot (x - x_1) \cdot \ldots \cdot (x - x_n)$ of a polynomial function g of degree n with roots $x_1, \ldots, x_n$;
- set up a linear system of equations for the coefficients of a polynomial function based on its properties.

2.5 Exponential functions: $f(x) = a \cdot b^x$ bzw. $f(x) = a \cdot e^{\lambda \cdot x}$

Students should be able to

- determine whether exponential functions are suitable to model a given process; recognize and sketch graphs of exponential functions;
- find the parameters a, b, c, and λ of functions of the type $f(x) = a \cdot b^x + c$ or $f(x) = a \cdot e^{\lambda \cdot x} + c$; interpret the parameters a, b, c, and the sign of λ;
- use exponential functions with $f(0) = a$ and $f(x + 1) = b \cdot f(x)$ or $f(x + 1) = e^{\lambda} \cdot f(x)$ to model processes of growth or decline; find and interpret the doubling time respectively the half-life of such models.

2.6 Sine function and cosine function

Students should be able to

- recognize and sketch graphs of functions of the type $f(x) = a \cdot \sin(b \cdot x) + c$ or $g(x) = a \cdot \cos(b \cdot x) + c$; find and interpret the parameters a, b, and c of such functions.

3 Analysis

3.1 Limits, difference quotients, and derivatives

Students should be able to

- find and interpret limits using limit laws and properties of elementary functions, based on an intuitive understanding of limits;
- use the relationship between difference quotients and the derivative; find difference quotients and derivatives; interpret and use difference quotients as slopes of secants and derivatives as tangent slopes.

3.2 Derivative functions and anti-derivatives

Students should be able to

- find the derivative function of power functions, polynomial functions, exponential functions, sine functions, and cosine functions; apply the constant multiple rule, the sum rule, the product rule, and the chain rule to find the derivatives of compositions of such functions;
- find antiderivatives of polynomial functions; find antiderivatives by reversing differentiation rules;
- apply the relationships between a function and its derivative functions as well as its antiderivative, also in graphical representations;
- describe the behavior of a function using its derivatives; find and interpret local extreme points and turning points; set up and interpret the equations of tangent lines.

3.3 Definite integrals

Students should be able to

- set up and interpret definite integrals as limits of sums of products in geometric and physical contexts; estimate the value of definite integrals using geometric considerations;
- determine the value of definite integrals using antiderivatives; recognize and use integral functions, i.e., $F(x) = \int_a^x f(t)dt$, as special antiderivatives of functions f.

4 Probability and statistics

4.1 Descriptive statistics

Students should be able to

- interpret graphical representations and diagrams (column chart, bar chart, line chart, pie chart) of data; supplement missing values in graphical representations and such diagrams;
- interpret and create tables, especially two-way tables; supplement missing values;
- find and interpret absolute and relative frequencies as well as statistical measures (arithmetic mean, median, mode, range, standard deviation); assess the impact of changes in the data on these statistical measures.

4.2 Combinatorics, random experiments, probability

Students should be able to

- apply the product and sum rules to solve counting problems; apply and interpret binomial coefficients in counting problems; find the number of permutations with or without repetitions;
- describe the sample space of random experiments verbally; describe events and complementary events verbally; represent events and complementary events as sets;
- interpret relative frequencies as estimates of probabilities; interpret probabilities as predictors of relative frequencies; find and interpret probabilities using the Laplace assumption; justify whether the Laplace assumption is admissible;
- use tree diagrams to model multi-stage experiments; find and interpret probabilities using the multiplication rule and the addition rule;
- determine and interpret conditional probabilities in connection with tree diagrams and two-way tables (Bayes' theorem is not required).

4.3 Random variables, binomial distribution, normal distribution

Students should be able to

- find and apply the probability distribution of a discrete random variable; find and interpret the expected value of a discrete random variable;
- use the binomial distribution and interpret quantiles;

- work with the normal distribution; interpret probabilities as areas under the graph of the density function and vice versa; relate the expected value and the standard deviation of a normally distributed random variable to the graph of its density function and vice versa;
- determine and interpret probabilities, quantiles, and intervals symmetric around the expected value of a normally distributed random variable; apply the empirical rule for one, two, or three standard deviations.